STUDENTS' MISTAKES IN ARITHMETIC OPERATIONS: HOW DO STUDENTS REASON?

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Abstract: *There are still many students, especially elementary school students, who make mistakes in arithmetic operations of integers. Of course, this must be helped immediately so that students do not experience further difficulties in learning mathematics. To help students correct these mistakes, it is important for teachers to know, not only the causes, but also how students reason during the arithmetic operations. This exploratory research seeks to trace the reasoning of students who have made mistakes in the arithmetic operation. Five students from grade 4 elementary school who made mistakes in an arithmetic operation test were interviewed to explore the causes of mistakes and their reasoning process. It was found that generally the cause of the mistakes was the limited knowledge of the prerequisites for performing arithmetic operations. Nonetheless, some traces of the students' reasoning showed noteworthy creative thinking. Teachers can use this reasoning trail to improve students' reasoning processes in arithmetic operations.*

Key Words*:* students' mistakes; students' errors; arithmetic operations; students' mathematical reasoning; reasoning processes.

Introduction

There is no doubt that mathematics is important for life and encourages the development of science and technology (Hodaňová & Nocar, 2016). Furthermore, Claessens & Engel (2013) said that it is important to teach mathematics early on. Therefore, it is not surprising that mathematics is taught from elementary school to university, even, informally, from early childhood education.

At the elementary school level, the mathematics learned is more about numbers. This is an important concept underlying the development of advanced mathematics. In mathematics, toeri about numbers is often called arithmetic. In arithmetic, the important and fundamental thing that must be mastered is the basic operations of arithmetic, namely addition, subtraction, multiplication, and division.

In learning arithmetic operations, students often get caught up in procedural knowledge, such as the use of the save-technique on addition or the borrow-technique on subtraction, without understanding how those techniques work. Of course, this is not good. It is important to understand that the basic operations of arithmetic will be well mastered if they have enough understanding of the concept of numbers, the concept of place-value, including understanding the meaning of addition, subtraction, multiplication and division. It is unfortunate that these prerequisites have received little attention from some teachers.

Today, there are still many students, especially elementary school students, who have difficulty in arithmetic operations. It has been widely reported by researchers (Fatimah et al., 2020; Helmy et al., 2018; Maelasari & Jupri, 2017). This situation certainly cannot be allowed. Attempts to improve the knowledge and skills of students in arithmetic operations should be made. It is necessary so that students do not have difficulty learning the next mathematics. As it has been known that mathematics is knowledge of a hierarchical nature (Hart, 1981).

In order for help and scaffolding to be provided appropriately, it is important for the teacher to know, not only the factors causing the student's mistakes, but also understand how the student made reasoning so as to make these mistakes. Therefore, this study aims to describe the reasoning of students who made mistakes in arithmetic operations and find their causes.

Method

This is exploratory research with qualitative approach. Five of the 36 elementary school students who were tested for the ability to perform arithmetic operations on integers were selected to be participants in this research. The five students were those who made mistakes in the arithmetic operation test.

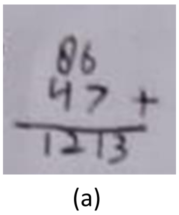
The participants were interviewed based on test results to explore the causes of errors and their ways of reasoning. Data from interviews supported by observations and answer sheets were analyzed qualitatively, including data analysis, categorization, reduction, display, and interpretation.

Results and Discussion

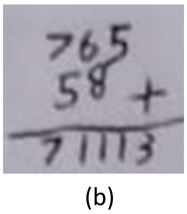
The following are the results and discussion which are divided based on the types of basic arithmetic operations, namely addition, subtraction, multiplication, and division, respectively.

Addition

Participant P1 performs the addition operation as shown in Figure 1. It is noticed that in both images, (a) and (b), P1 makes the same mistake. He said that he added ones with ones, 6 + 7 = 13, and added tens with tens, 8 + 4 = 12, next he wrote down the results below the line to 1213. It is clear that he made the mistake of placing 2 digits of tens, that is, digits 1 of 13 and digit 2 of 12, on different places. We tried to get deep, why did he do that?



(a)



(b)

Figure 1. P1's work in addition

We begin with the question, “does he understand the value of places from 86?” He answered, “86 = 8 tens + 6 ones.” We continue by asking the meaning of 47. Again, he answered correctly that 47 = 4 tens + 7 ones. Here, it seems that he understands about the concept of place-value. Why do you add 6 with 7, and 8 with 4? He replied that ones are added with ones, as are tens with tens. What does 13 means in the result of adding 6 + 7? What does 12 means in the result of adding 8 + 4? He explained, "6 and 7 are ones, the result of addition is 13 ones; 8 and 6 are tens, hence the result of the addition is 12 tens." So what does 1213 mean in the result of the addition? "12 tens + 13 ones." Shouldn't it be 1213 = 1 thousands + 2 hundreds + 1 ten + 3 ones? At this point he began to be confused.

By utilizing P1's reasoning processes, we helped him to make improvements. It is started by giving the understanding that 13 ones = 1 tens + 3 ones, and 12 tens = 1 hundreds + 2 tens, so that the addition becomes

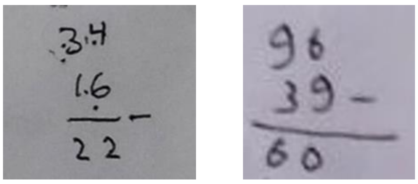
86 = 8 tens + 6 ones
<u>47 = 4 tens + 7 ones +</u>
= 12 tens + 13 ones = 1 hundreds + 2 tens + 1 tens + 3 ones
= 1 hundreds + (2 + 1) tens + 3 ones
= 1 hundreds + 3 tens + 3 ones = 133.

In the composed addition form is written

86
<u>47 +</u>
13 □ 6 + 7
8 + 4 □ <u>12 +</u>
133.

Subtraction

The results of the interviews on P1 and P2 showed a mistaken understanding of the subtraction operation. They assume that a number cannot be subtracted by a larger number. As a result, in subtraction 34 – 16, P2 subtracts 6 by 4 (see Figure 2(a)), while P1, in subtraction 96 – 39, assumes that 6 – 9 = 0 (see Figure 2(b)).

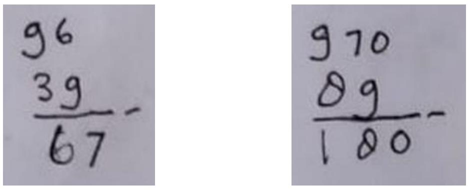


(a) (b)

Figure 2. (a) P2’s work and (b) P1’s work in subtraction

Elsewhere, P3 shows a better understanding, but P3 is wrong in applying the borrow-technique (Figure 3). Another mistake he made was the inaccuracy in putting 89 in subtraction 970 – 89 which showed a lack of understanding of the concept of place-value (Figure 3(b)).

Overall, the three participants did not understand the meaning of subtraction in integers and did not understand the concept of place-value that 1 tens could mean 10 ones, so that they have been wrong in applying the borrow-technique in subtraction.



(a) (b)

Figure 3. P3's work in subtraction

To correct this, we give them an understanding that 1 tens = 10 ones, so that, in the case of a subtraction of 96 – 39, the subtraction becomes

96 = 9 tens + 6 ones = 8 tens + 16 ones
39 = 3 tens + 9 ones = 3 tens + 9 ones -
5 tens + 7 ones = 57,

before using the compound subtraction form.

In this way, we give an understanding that borrowing 1 out of 9 by 6 means borrowing 1 tens = 10 ones out of 9 tens, which is added to 6 so that it becomes 16 ones. This at once turned the 9 tens into the 8 tens. Thus, they not only know the procedure for subtraction and using the borrow technique, but understand how this technique works.

Multiplication

In Figure 4, P4 performs an incomplete multiplication procedure. P4 has done so as follows. 2 · 6 = 12, write 2 save 1; 1 · 6 = 6, plus 1 equal to 7; 1 · 1 = 1; so that the result became 172. P4 forgot to do 2 · 1.

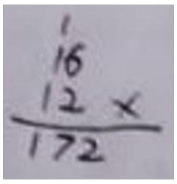


Figure 4. P4's work in multiplication

In other places, P5 performs the multiplication procedure completely and uses the save technique correctly. But, P5 is not right in placing 26 as a result of 1 tens multiplied by 26. This suggests that P5 lacks understanding of the place-value in multiplication (Figure 5).

In general, both participants had knowledge of the long-arranged multiplication procedure and were able to use the save-technique in multiplication. Nevertheless, these knowledges are not supported by a good understanding of the place-value.

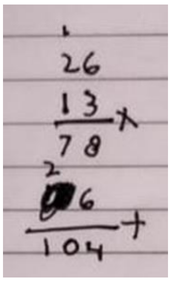


Figure 5. P5’s work in multiplication

We propose that before students use the arranged multiplication procedure, students are invited to learn and understand multiplication by utilizing multiplication properties, such as the distributive property of multiplication over addition. For example,

$$16 \cdot 12 = 16 \cdot (2 + 10) = (16 \cdot 2) + (16 \cdot 10) = 32 + \mathbf{160} = 192.$$

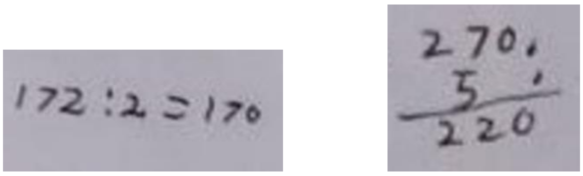
If it applied in the long-arranged multiplication, it is obtained

$$\begin{array}{r} 16 \\ \underline{12\ \cdot} \\ 32 \\ \underline{\mathbf{16}\quad +} \\ 192 \end{array}$$

In this way, students can understand why 16 is written somewhat to the left in a long-arranged multiplication, certainly because those 16 are 16 tens, which is 160.

Division

When we asked P4 how he did the division of 172 |2 (Figure 6(a)), he said, "I subtracted 2 from 172." He reasoned, "if I have 172 oranges and if I give 2 to my friend that means it subtracts 2 from my oranges?" It can be seen that P4 has a misunderstanding about the division operation. He equates the division and the subtraction operation. Surprisingly, when we give another task, which is 270 |5 (Figure 6(b)), he actually subtracts in the wrong way. "I can't subtract 0 by 5, so I subtract 7 by 5." It seems that he has not only difficulties with the division operation, but also with the subtraction operation.



(a) (b)

Figure 6. P4’s works in division

In other places, P2 has done the division in different ways for 28 |2 (Figure 7). He divided 8 ones by 2 i.e. 4 ones and divided 2 tens = 20 by 2 i.e. 10. But he miswritten the result 104 because he did not realize that 10 ones equal to 1 tens. He did not correctly understand the concept of the place-value in division.

In general, both, especially P4, do not understand the meaning of division as an inverse of multiplication and as a repeated subtraction. It is important for the teacher to teach the meaning of division as an inverse of multiplication and as a repeated subtraction. It is often found that teachers teach a standard-arranged division without strengthening students' understanding of the meaning of division.

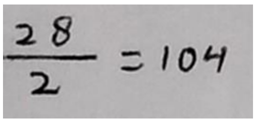
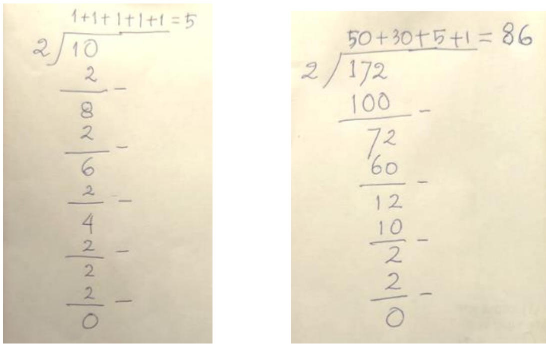


Figure 7. P2’s work in division

After strengthening students' understanding of division as the inverse of multiplication, such as 10 |2 = 5 because 5 · 2 = 10, teach students that 10 |2 = 5 because need to be 5 times subtracting 2 from 10 so that the remainder is 0. Informally, this can be done in a nonstandard-arranged division as shown in Figure 8 before students do with standard-arranged division.



(a) (b)

Figure 8. Nonstandard-arranged divisions

In general, students' mistakes in arithmetic operations are caused by limited knowledge of the prerequisites and lack of understanding of the meaning of the arithmetic operations. Therefore, before teaching arithmetic operations, it is important for teachers to check the adequacy of students' prerequisite knowledges. This is important, because students will only be able to learn well if they are in the zone of proximal development (Abtahi, 2017; Clapper, 2015; Eun, 2017; Margolis, 2020; Siyepu, 2013).

In addition, it is also important for teachers to familiarize students with reasoning and constructing their own mathematics, even though it is informal mathematics (Gravemeijer, 2020; Inglis & Foster, 2018; van der Beek et al., 2017). Sometimes students' creative thinking emerges when we give them the freedom to use their thinking to solve problems (Aini et al., 2020; Dhayanti et al., 2018; Hadar & Tirosh, 2019; Puspitasari et al., 2018). As shown by the participants in this research, although it is still rudimentary, but their creative thinking and reasoning process can be utilized by teachers to help them develop their knowledge and skills.

Conclusion

students' mistakes in arithmetic operations are caused by limited knowledge of the prerequisites and lack of understanding of the meaning of the arithmetic operations. Nonetheless, some traces of student reasoning show noteworthy "creative thinking". Teachers can use this line of reasoning to improve students' reasoning processes.

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